Math 1A: Calculus **Instructor: Alexander Paulin**

Quiz 6: §3.5-3.7

Your name: *Discussions 201, 203 // 2018-10-19*

Problem 1 (0 points per incorrect answer, 2 points per correct answer, and 1 point per unanswered). Determine whether each of the following statements is **True (T)** or **False (F)**, by circling the appropriate choice. A statement counts as false if *any* part of it is false. (If you do not know the answer, you should skip rather than guess.) No justification is needed.

T / **[F]** Let $f(x) = \ln x$. Then its derivative is $f'(x) = 1/x$ with domain $(-\infty, 0) \cup (0, \infty)$.

[T] / **F** The tangent line to the curve defined by the equation $y^3 = x^6$ at the point $(0, 0)$ is given by $y = 0$.

[T] / **F** Let $p(t)$ be the function recording the position of a particle moving along a line (e.g. the y-axis). Suppose that at time t_0, both $p'(t_0)$ and $p''(t_0)$ exist and moreover $p'(t_0) \cdot p''(t_0) < 0$. Then the particle is slowing down at time t_0.

Problem 2 (4 points, Stewart §3.6 Ex. 44). Compute $\frac{dy}{dx}$:

$$y = x^{\cos(x)}.$$

Solution: Logarithmic differentiation gives:

$$\ln y = \cos(x) \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = -\sin(x)\ln x + \cos(x)\frac{1}{x}$$

$$\frac{dy}{dx} = \boxed{x^{\cos(x)}\left[\frac{\cos(x)}{x} - \sin(x)\ln x\right]}.$$

□

(Continued on back of sheet.)

Problem 3 (3 + 2 = 5 points). Consider the curve defined by the equation

$$y^2 = 3x^2(x+1).$$

(a) Find the equation of the tangent line to the curve at the point $(2, -6)$.

(b) Find (the numerical value of) $\frac{d^2y}{dx^2}$ at the point $(2, -6)$.

Solution:

(a) Implicit differentiation gives

$$2y\frac{dy}{dx} = 9x^2 + 6x \tag{0.1}$$

so at the point $(2, -6)$ the slope of the tangent line is -4 (by solving the above). Hence point-slope form gives

$$y - (-6) = -4(x-2)$$

$$\boxed{y = -4x + 2.}$$

(b) Differentiating equation (0.1) again gives

$$2\left(\frac{dy}{dx}\right)^2 + 2y\frac{d^2y}{dx^2} = 18x + 6$$

and plugging in what we know at $(2, -6)$ makes this into the equation

$$2(-4)^2 + 2(-6)\frac{d^2y}{dx^2} = 18(2) + 6$$

for which one solves $\frac{d^2y}{dx^2} = \boxed{-5/6}$.

□